



## PORTFOLIO RISK MANAGEMENT WITH FINANCIAL STATISTICS APPLIED TO STOCK MARKET DATA

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### ABSTRACT

In this paper we present a systematic strategy of using statistical techniques for analysing investment in shares. We explore the building of efficient portfolios through optimization using real stock market data for three private limited companies in UK, Vodafone, Barclays and Next. We examine how basic statistical tools can assist an investor in diversifying his or her risk. Monthly historical data of shares for the five years from January 2006 to December 2010 are used for this analysis. Investing in stock markets bears some risk depending on the variance and return of the stock. We discuss the relationship between the average return and risk and also discuss how to minimize the risk of the portfolio. It appears that the Barclays is more riskier for investment than the other two companies. We also discuss why diversification works by identifying the efficient frontier and by constructing the portfolio possibilities curve. This study will assist investors in making more informed decisions regarding investing in stocks.

**Key words:** Frontier, Portfolio, Return, Risk, Stock



## 1. INTRODUCTION

The goal of performing fundamental financial statistical analysis with stocks data is to produce a value that an investor can compare with the stock's current price, with the aim of figuring out what sort of position to take with that stock. Shares also known as equities represent a person's ownership of a company. When shares are bought the person owns part of the company. These shareholders then become key decision makers when matters of the company are concerned. Small, medium and large organisations can have shareholders. Having shareholders allows a company to raise finance more widely as a company can become listed on stock exchanges if they wish to after an approval process. Many studies have ascertained shares (or equities) happen to be one of the most profitable long-term investments in the financial marketplace. However with shares like other financial assets come with risks, as prices can fluctuate but generally shares generate good long term returns. Shareholders get a return for their money through dividends. Using investment analysis we evaluate and explain the construction and management of portfolios through various techniques and methods. There are many studies that work on portfolio ([1], [2], [3], [4], [5]). This paper provides analysis of closing share prices of three companies listed in the London stock exchange- Vodafone Plc [6], Barclays Plc [7] and Next Plc [8]. Such data is openly available at online sources such as [www.finance.yahoo.com](http://www.finance.yahoo.com).

Vodafone is a UK based global telecommunication company. The company is registered in London stock exchange. On 1<sup>st</sup> January 1985 Vodafone was the first company which made a mobile call. Within fifteen years the Vodafone network has become the largest telecommunications company in Europe. They are providing their telecommunication services in 21 countries. Barclays have over 300 years of history and experience in the field of banking. Barclays operates in more than 50 countries and have more than 145,000 employees worldwide. They are moving, lending, investing and protecting the money of the customers and their clients worldwide. Barclays is the 10<sup>th</sup> largest banking and financial service and 21<sup>st</sup>

largest company in world [9] According to their annual report 2010, total income was £4518m in UK from retail banking which is 22% higher than 2009. Barclays is listed in London stock exchange. Next Plc. is nearly 150 year old and the Next brand was created in 1982. They deal in retail business and offer men, women and children's clothing, accessories. The retail chain was launched in February 1982. Next is running their retail stores in more than 35 countries.

## 2. SHARE PRICE TREND

The following graphs (Figures 1-3) show the fluctuations in the closing price of shares for the three companies over the period January 2006 – December 2010. During mid-2006 Vodafone experienced a slow

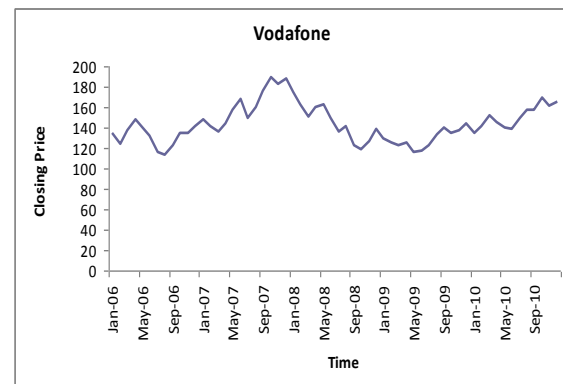


FIGURE 1: Closing share prices of Vodafone

but steady increase in its share prices; this was until the last quarter of 2007, where its share prices were at their highest during this period. However during 2008 Vodafone experienced a consistent fall in its share prices for the period of the year 2008. This was due to the global financial crisis. For the period between last quarter of 2008 and mid-2009 the company experienced relatively stable share prices. From this point onwards until the end of 2010 Vodafone's share prices increased relatively slowly.

The graph shows in the beginning quarter of 2006 the share prices of Barclays has increased, there was then a sharp drop in the share price in the middle



of 2006. From mid-point of 2006 Barclays experienced a slow increase in its share prices until the beginning of 2007. From this period onwards there was a consistent fall in share prices for two years this is due to the global financial crisis. From the first quarter of 2009 share prices increased slowly but steadily. But since this period share prices fluctuated several times this unpredictability was consistent till the end of the period.

Despite Next having large increments in its share price in 2006, share prices fell sharply in the last quarter of 2007, through to the beginning of 2008. Share prices were less than half compared to previously in this period. From this period onwards share prices increased drastically to the similar prices at the end of the third quarter of 2007. At the end of this period share prices fluctuated at an unpredictable rate.

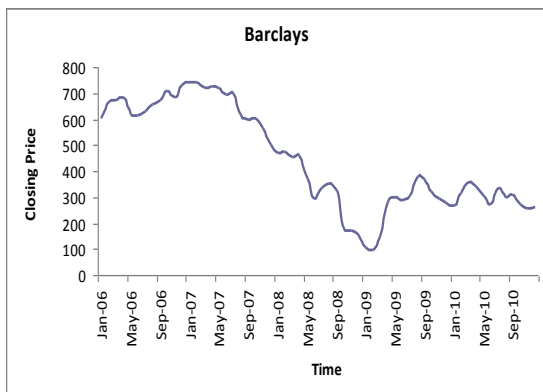


FIGURE 2: Closing share prices of Barclays

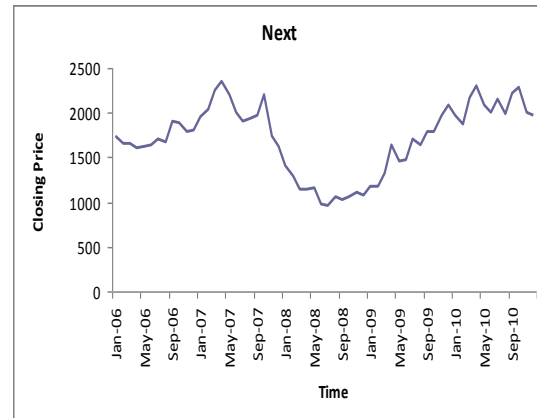


FIGURE 3: Closing share prices for Next

### 3. METHODS AND ANALYSIS

#### 3.1. Basic Analysis

First we calculate the simple return for the chosen companies. The formula used to define the return at time  $t$  of the stocks is as follows. Following histograms show the returns of three stocks in Figure 4.

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

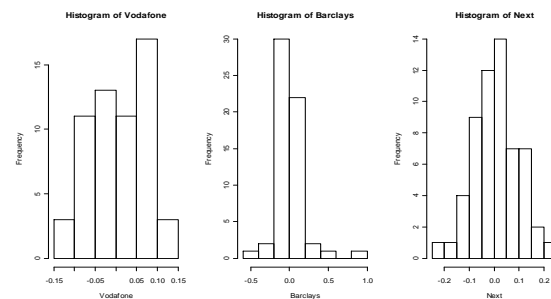


FIGURE 4: Histogram of the return of the three stocks Vodafone and Barclays and Next over time

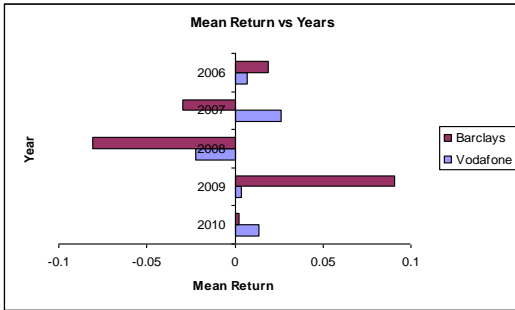


Figure 5: Mean return vs years for Vodafone and Barclays

We calculate the mean and variance of the returns of stock *i* as

$$\bar{R}_i = \frac{1}{n} \sum_{t=1}^n R_{it}, \sigma_i^2 = \frac{1}{n-1} \sum_{t=1}^n (R_{it} - \bar{R}_i)^2$$

and the covariance between the returns of stocks *i* and *j* is defined as

$$cov(R_i, R_j) = \sigma_{ij} = \frac{1}{n-1} \sum_{t=1}^n (R_{it} - \bar{R}_i)(R_{jt} - \bar{R}_j)$$

Yearly average return for Barclay as shown in Figure 5 is always higher than that of Vodafone. Besides, Barclays has very high negative return in two consecutive years 2007, 2008. Vodafone has negative average return only for one year 2008 and which is much less compared to that of Barclays. In terms of yearly median return as shown in Figure 6, Barclays tend to have more negative median return year whereas Vodafone has only one negative median year as similar to the number of negative mean year.

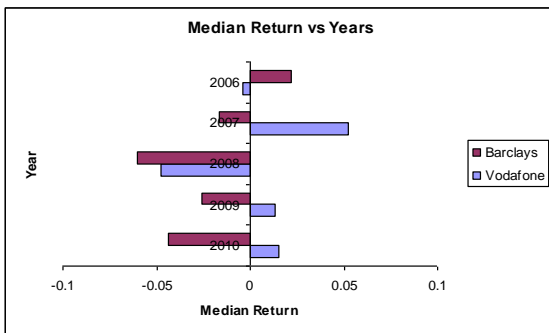


FIGURE 6: Median return vs years for Vodafone and Barclays

3.2. Risk Analysis

Vodafone has low risk but higher return whereas Barclays has the opposite low return but higher risk. So, it is suggested that it is not true the higher the risk the higher the return. It could be any possibilities in practice.

TABLE 1: Mean and Risk (standard deviation) of return

	Average Return	Risk
Vodafone	0.0056	0.0651
Barclays	0.000047957	0.1827781

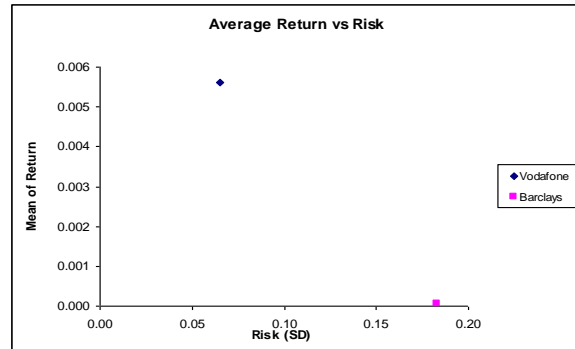


FIGURE 7: Average return against risk

The covariance matrix of share prices shows the relationship among the companies. It also assumes that the returns on risk factors are normally distributed.

TABLE 2: Variance-Covariance matrix of share prices

	Vodafone	Barclays	Next
Vodafone	0.00424		
Barclays	0.001645	0.033408	
Next	0.001745	0.006105	0.007528

If we take first two stocks which is Vodafone and Barclays so it appears from the analysis that the



risk is the lowest for Vodafone so investor would be interested to invest firstly into Vodafone and then in Barclay. If we take three companies including Next as well then It appears from the analysis that the risk is the lowest for Next and so why investor would be interested to invest firstly into Next and then possibly into Vodafone and then into Barclays.

**3.3. Optimization of the Risk of the Portfolio with Two Stocks**

Suppose  $R_A$  and  $R_B$  be the returns of stocks A and B respectively, and if  $x_A$  and  $x_B$  be the proportions of the available funds invested in each of the stocks. Then the portfolio can be defined by  $x_A R_A + x_B R_B$ . Now if the variance of the portfolio is minimized it mean risk of the portfolio is minimized therefore it is important the investor needs to minimize the variance of the portfolio stocks. In other words, he will seek for minimizing the variance of the portfolio i.e.

$$\text{Min } \text{Var}(x_A R_A + x_B R_B) = \text{Min } x_A^2 \text{Var}(R_A) + x_B^2 \text{Var}(R_B) + 2 x_A x_B \text{Cov}(R_A, R_B)$$

which is subject to the budget constrained  $x_A + x_B = 1$ . Hence it becomes

$$\text{Min } x_A^2 \text{Var}(R_A) + (1 - x_A^2) \text{Var}(R_B) + 2 x_A (1 - x_A) \text{Cov}(R_A, R_B).$$

In order to optimize the above equation we differentiate it with respect to  $x_A$  and then equates to zero. This results in the solution for  $x_A$  as below

$$x_A = \frac{\text{var}(R_B) - \text{cov}(R_A, R_B)}{\text{var}(R_A) + \text{var}(R_B) - 2\text{cov}(R_A, R_B)}$$

and so

$$x_B = \frac{\text{var}(R_A) - \text{cov}(R_A, R_B)}{\text{var}(R_A) + \text{var}(R_B) - 2\text{cov}(R_A, R_B)}$$

We first consider two stock Vodafone and Barcklays. Now using the above formula and the variance and

covariance estimates as shown in Table 2 we calculate proportions of the available funds invested in Vodafone and Barcklays as below.

$$x_{Vodafone} = (0.033408 - 0.001645) / (0.00424 + 0.033408 - 2(0.001645)) = 0.92447$$

and so

$$x_{Barcklay} = 1 - 0.92447 = 0.0755282$$

Therefore if the investor invests 92% of the available funds into Vodafone and the remaining 8% into Barcklays and the minimized variance of the portfolio would be

$$\text{Var}(0.92447 R_{Vodafone} + 0.07552 R_{Barcklay}) = 0.00404.$$

The corresponding expected return of this portfolio is

$$E(R_{portfolio}) = E(0.92447 R_{Vodafone} + 0.07552 R_{Barcklay}) = 0.92447 E(R_{Vodafone}) + 0.07552 E(R_{Barcklays}) = 0.00518$$

So, still people would be interested to invest for Vodafone and not for Barcklays. So we can say that if investor is interested in spending two companies then 92.4% Vodafone and 7.6% Barcklays give less risk than the individual stock. In Table 3 we estimate the expected portfolio and minimized risk of the portfolio for different combination of proportions of the available funds invested in Vodafone and Barcklays.

TABLE 3: Expected return and risk of portfolio for Vodafone and Barcklays

$x_{Vodafone}$	$x_{Barcklays}$	$E(R_{portfolio})$	Risk
1	0	0.00560	0.06512
0.9	0.1	0.00504	0.06375
0.8	0.2	0.00449	0.06765
0.7	0.3	0.00393	0.07599
0.6	0.4	0.00338	0.08752
0.5	0.5	0.00282	0.10116
0.4	0.6	0.00227	0.11616
0.3	0.7	0.00171	0.13205
0.2	0.8	0.00116	0.14857



0.1	0.9	0.00060	0.16551
0	1	0.00005	0.18276

We observe different combination of Vodafone and Barclays and their risk and expected return. So this combination shows that it is good for investor to invest more in Vodafone and less on Barclays. We now plot the expected return versus the risk in order to generate the portfolio possibilities curve (Figure 8). Here, the efficient frontier is the area between the top two points shaded by red colour as there the risk is lowest and the return is the higher (area shown by the connected line).

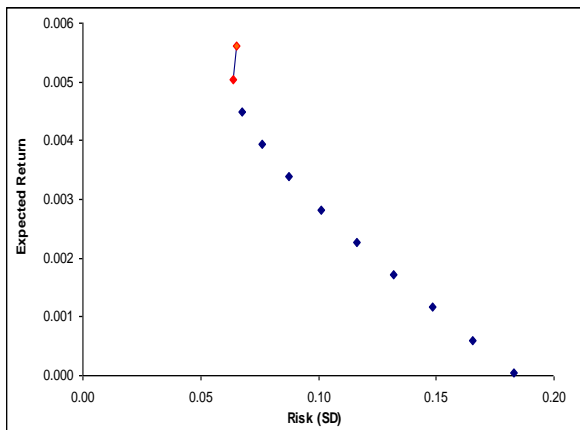


FIGURE 8: Portfolio possibilities curve for two stocks Vodafone, Barclays

### 3.4. Optimization of the Risk of the Portfolio with Three Stocks

Using three stocks the minimum risk portfolio can be obtained in the similar procedure as that is done with using two stocks. We need to find

$$\text{Min } \text{Var}(x_A R_A + x_B R_B + x_C R_C)$$

Or

$$\begin{aligned} \text{Min } & x_A^2 \text{Var}(R_A) + x_B^2 \text{Var}(R_B) + x_C^2 \text{Var}(R_C) \\ & + 2 x_A x_B \text{Cov}(R_A, R_B) \\ & + 2 x_A x_C \text{Cov}(R_A, R_C) \\ & + 2 x_B x_C \text{Cov}(R_B, R_C) \end{aligned}$$

with subject to  $x_A + x_B + x_C = 1$ . The resulting equation is a quadratic equation and it is not possible to solve using simple approach. There are many statistical software to solve such quadratic equation. However it can be solved through quadratic programming [10]. We use different combination of values for  $x_{Vodafone}$ ,  $x_{Barcklay}$  and  $x_{Next}$  that correspond the proportion of the available funds invested into Vodafone, Barcklays, and Next respectively. We then make portfolio possibilities curve (Figure 9).

It is very difficult to find out where is exactly the efficient frontier with a small number of combinations of the proportions of the funding invested for three stocks. In this graph the efficient frontier is that connects the minimum risk portfolio to the maximum return portfolio. It seems that the efficient portfolio region takes place in the left top of the graph which could be identified clearly if we take all possible combination of the proportions of the funds invested.

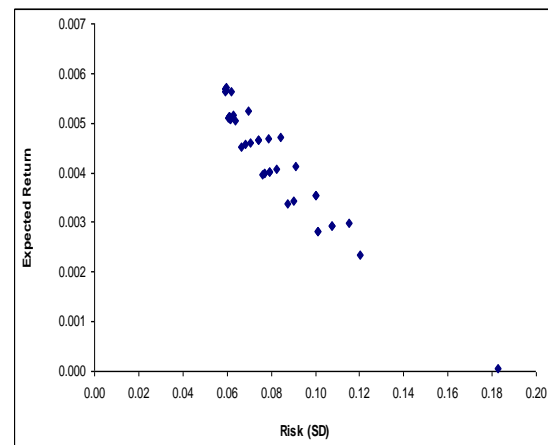


FIGURE 9: Portfolio possibilities curve for stocks Vodafone, Barclays and Next

### 4. CONCLUSION

This study serves as an introduction and demonstration of statistical finance theories and their application with a simple user friendly stock investment portfolio. We use three real datasets from



three famous companies working in UK- Vodafone, Barclays, and Next listed in the London stock exchange. Our findings suggest that out of the three companies from an investor's view when the three stocks are concerned it is better to invest in shares of Next, because variance of its return was lower than the other two companies. When two stocks are considered, it is evident that investors would want to invest more in Vodafone stock than Barclays in order to minimize the risk of the portfolio. Barclays presented the highest risk as variance of its return was much higher. So, the portfolio selection and risk management framework discussed in this study will assist the investors to take their decision about the future risk of investments.

#### REFERENCES

- [1] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, Issue 1, pp. 77-91.
- [2] Elton, Edwin J., Gruber, Martin M. and Padberg, Manfred W. (1976). Simple Criteria for Optimal Portfolio Selection. *The Journal of Finance*. 31(5), 1341-1357.
- [3] Elton, Edwin J., Gruber, Martin M. and Padberg, Manfred W. (1977). Simple Rules for Optimal Portfolio Selection: The Multi Group Case. *The Journal of Financial and Quantitative Analysis*. 12(3), 329-345.
- [4] Kwan, Clarence C.Y. (1984). Portfolio Analysis Using Single Index, Multi Index, and Constant Correlation Models: A Unified Treatment. *The Journal of Finance*. 39(5), 1469-1483.
- [5] Christou, N (2008). Enhancing the Teaching of Statistics: Portfolio Theory, an Application of Statistics in Finance, *Journal of Statistics Education*, vol.16, no.3
- [6] Vodafone PLC, Available at [http://vodafone.com/content/dam/vodafone/investors/annual\\_reports/](http://vodafone.com/content/dam/vodafone/investors/annual_reports/)
- [7] Barclays PLC, Available at <http://uk.finance.yahoo.com/=Barclays>.
- [8] Next PLC, Available at <http://uk.finance.yahoo.com/=Next>
- [9] Forbes magazine (2010), investing in financial companies, available at [www.forbes.com/markets](http://www.forbes.com/markets)
- [10] Bazarrá, M., Shetty, C, (1979). Nonlinear Programming Theory and Algorithms, Wiley