International Journal of Information Technology and Business Management
29th July 2014. Vol.27 No.1

ISSN 2304-0777 www.jitbm.com

RESOLUTION OF CREDIT RISK OPTIMIZATION PROBLEMS USING THE MONTE CARLO SIMULATION

João L. Chela, Luiz L. Salles-Neto , Antonio A. Chaves & Renato Cesar Sato
Science and Technology Department, ICT, UNIFESP.
13084-790, São José dos Campos, SP
E-mail: chela.risco@gmail.com, luiz.leduino@gmail.com, antoniochaves@gmail.com, rcsato@gmail.com.

Abstract:
In recent years the financial industry in Brazil and in the world has been modernized focusing on using more robust and sophisticated mathematical techniques at the moment of strategic decisions. A common strategic decision in financial institutions is the optimal allocation of financial assets. In general, this problem consists of allocating an amount of these assets in the different investment options available, so that when given an expected return, it minimizes the risk of the investment portfolio. In this case, the main types of financial risks that predominate in an investment portfolio are market and credit risk. The purpose of this paper is to propose an implementation on the resolution of the credit optimization problem using the Monte Carlo simulation.

Keywords: *Credit risk, Optimization, Monte Carlo Simulation*

1 Introduction

Every day many financial institutions in Brazil and around the world are faced with strategic decisions related to credit risk management, see [1.7]. A strategic decision involves the optimal selection of portfolios or investments subject to credit risk. On January 19th, 2014 the newspaper *Folha de São Paulo* reported that an operation of the bank BVA affected more than 70 funds:

"More than 70 pension funds of state-owned companies and city halls around the whole country run the risk of losing most of the R$ 2.7 billion that they had invested in the purchase of securities backed by loans originating from the BVA Bank, which has been in a process of liquidation since August".

It is in fact, an important question for the country and its population, not only for the financial institutions.

In the optimal selection of investments with credit risk, in general, the financial institution's asset manager needs to make the decision to invest in assets for its clients or for the financial institution itself. If investment decisions involve fixed income securities with credit risk, in this case, private securities, the manager will need to make sure that the portfolio maintains an expected return and offers the minimum credit risk.

The decision above involves the solution of an optimization problem that often presents complex theoretical properties and deserves attention in its computational treatment. Moreover, the generation of credit scenarios that can leave the problem with a computational complexity is necessary.

Methodologies and alternatives of the associated problem solution are stated in literature, see [6.8.3]). In this context, this paper proposes an optimization approach of credit risk using the Monte Carlo simulation, which can be applied by using only a few computational and financial resources in some cases.

2 Credit Risk Measures

2.1 Credit Risk

According to resolution CMN 3721, published by The Brazilian Central Bank on April 30th, 2009, we can define credit risk as the possibility of losses occurring associated to not the fulfillment by the borrower or customer of its respective financial obligations in the terms agreed to.

International Journal of Information Technology and Business Management
29th July 2014. Vol.27 No.1

ISSN 2304-0777 www.jitbm.com

2.2 Probability of Default

The probability of default is characterized as the percentage that corresponds to the long-term expectation of default rates for the time horizon (e.g. 1 year). The probability of default has an important role in the management of credit risk, aiding in the constitution of provisions, the pricing of loans and establishing credit limits.

2.3 Loss of Credit

In this paper, we shall call loss of credit, or simply exposure to credit risk, the financial amount that a financial institution loses, given that default of a client occurs. For example, if a customer took out a loan of 1,000 (One thousand reais) and is in default, the bank can lose the amount of 1,000.

3 Monte Carlo Simulation for Credit Risk

A Monte Carlo method (MCM) is any method of a class of statistical methods based on massive random sampling for obtaining numerical results.

In the case of the Monte Carlo simulation for credit risk, we are interested in simulating a variable that receives the value 0 (zero) or 1 (one).

To illustrate the simulation, consider a credit portfolio composed of two clients, each client has default probabilities (p_A and p_B), loss of credit (E_A and E_B) for each client we will associate a variable z which receives the values 0 (zero) or 1 (one), and this variable will be assigned the value of 1 (default) with probability p e 0 (No Default) with probability (1-p), this variable has Bernoulli distribution with average p and variance $p-p^2$ see [5].

Therefore, each client will have a variable that will receive these values and be associated with their respective probabilities of Default.

Now we will simulate default scenarios for our portfolio. A client will be in default conditions if the simulated variable receives the value 1.

To generate these scenarios, consider the following algorithm:

N being the number of random scenarios that we are interested in generating. Step 1: (i = 1::::; N)

1_A: (For the client A) - Generate a random number with uniform distribution between [0; 1], in this case μ_A.

If $\mu^i_A < p_A \rightarrow z^i_A = 1$(Client default), or on the contrary $z^i_A = 0$(No default).

1_B: (For the client B) - Generate a random number with uniform distribution between [0; 1], in this case μ_B. If $\mu^i_B < p_B \rightarrow z^i_B = 1$(Client default), or on the contrary $z^i_B = 0$(No default).

Repeat the procedure for the N scenarios and consider the following vectors.

$z^t_A = [z^1_{A,\ldots,} z^N_A]$ and $z^t_B = [z^1_{B,\ldots,} z^N_B]$.

Note that the components of these vectors are zero and one. In this manner, we generate N credit scenarios for the portfolio.

For example, supposing that for the scenario k, $z^k_A = 1$ e $z^k_B = 0$, this scenario signifies that client A defaulted, but client B did not.

The scenarios being generated, we have to define a loss function given the scenario carried out.

If $f(x; y_i) = x\, y^t_i$, where $x = [E_A \;\; E_B]$ e $y_i = [z^i_A \;\; z^i_B]$.

For example, if x = [1:000; 500] and y_i = [1; 0], then $f(x; y_i)$ = 1:000 x 1 + 500 x 0 = 1:000.

Therefore the loss in this portfolio will be 1,000. The function $f(x; y_i)$ shall be called loss function evaluated in x in the scenario y_i and will be used in the next section.

Note that the above procedure is simple and can be generated using common computer tools, for example Excel.

This and other Monte Carlo simulation techniques can be found in [7].

4 The Problem of Credit Risk Optimization

ISSN 2304-0777 www.jitbm.com

The objective of this section is to present the credit risk optimization problem of an investment portfolio studied in [3]. For this, we need to define some risk measures.

If we consider f : IR^n x IR^m → IR as a loss function for a credit portfolio, x ∈ X in IR^n is the portfolio composition and y ∈ IR^m is a random vector of credit scenarios. In this way, if y ∈ IR^m is a random vector, we can associate with it a function of probability distribution p : IR^m → IR, and therefore define the probability that the loss would not be greater than a scale in the following form.

$$\varphi(x,\alpha) = \int_{f(x,y)\leq\alpha} p(y)dy$$

$\varphi(x,\alpha)$ Represents the probability of the loss of the portfolio being less or the same as a. In this manner, we can define the first risk measurement.

We consider the following function

$$\alpha = \alpha(x,\beta) = \min\{\alpha \in \text{IR} : \varphi(x,\alpha) \geq \beta\}$$

For x fixed, the amount $\alpha(x,\beta)$ will be called Value at Risk, or simply βVaR associated to the confidence interval β.

As well as the βVaR, we can define a risk measure conditional to the VaR which will be called βCVaR(Conditional VaR), this measure is the average value of the loss amount which is, mathematically speaking, greater or equal to the βVaR:

$$\theta(x) = (1-\beta)^{-1}\int_{f(x,y)\leq\alpha} f(x,y)p(y)dy$$

The next theorems are important to relate the minimizers of the function defined above and were demonstrated in [6].

Theorem 4.1. For x ∈ IR^n , we suppose that $F_\beta(x,\alpha)$ is related, continuous and differentiablein relation to α. So βCVaR associated to any x ∈ Rn fixed can be determined in the following way:

$$\theta(x) = min_\alpha F_\beta(x,\alpha)$$

Proof: see [6].

Note that the theorem above affirms that the βCVaR can be calculated resolving a convex optimization problem without the βVaR being calculated *a priori*.

Theorem 4.2. We suppose that $F_\beta(x,\alpha)$is related, continuous and differentiable. Therefore, the minimum βCVaR associated to any x ∈ IR^n can be calculated minimizing $F_\beta(x,\alpha)$ in relation to (x;α), that being:

$$min\ \theta(x) = min_{(x,\alpha)} F_\beta(x,\alpha)$$

Proof: see [6].

The theorem above characterizes the problem of credit optimization taking as its basis the objective function βCVaR. Note that even if an optimal portfolio composition is used that minimizes the βCVaR, this solution does not minimize the βVaR, but it is the βCVaR correspondent for this investment composition x. In [6] is proposed the following approximation of the objective function of the problem.

$$F(x,\alpha) = \alpha + \frac{1}{(1-\beta)*J} \times \sum_{i=1}^{J}[f(x,y_i) - \alpha]^+$$

Where J is the amount of random scenarios, y_i it is a vector of scenarios and $f(x,y_i)$ the sum of the loss in each scenario give the composition x.

Therefore, we can use this new discrete objective function for the implementation of the optimization problem.

Some characteristics of this problem are highlighted in [6].

ISSN 2304-0777 www.jitbm.com

1- The discretized problem becomes a linear program problem if $f(x, y_i)$ are linear and considering that the set of restrictions is linear.

2- The objective function is not differentiable.

3- The objective function F is convex when $f(x, y_i)$ are convex.

5 Numerical Tests

We performed numerical tests considering the generation of 2,000 (for 4 investiment options) and 10,000 (for 10 options) credit scenarios, i.e., for every problem were generated f(x; y_i) ; i = 1 :::; 2,000 (or 10, 000) as proposed in The Monte Carlo simulation section. All simulations were performed on Excel. We used the optimization tool of Microsoft Excel (SOLVER) with the model GRG (Generalized Reduced Gradient) and did not use any differentiable reformulation for the objective function.

Therefore, we consider the following example:

1 - A financial institution intends to invest 10M (ten million reais) in fixed income securities which are subject to credit risk, but to do this it needs to decide how this investment should be made so that it minimizes the βCVaR and obtains a minimum desired return. We use β= 99%.

The tables below describe the characteristics of each investment option (fixed income security), where for each option we have: PD: Probability of Default of the Issuer's Security; spread: Return in the period (year) of the Security; Limit: Maximum percentage of the investment option.

In this case, the optimization problem will have the following constraints:

1 - The percentages of investments (Weights): the variable x must be positive and not greater than 100%, or concentration limits.

2 – Minimum return required in this case $\sum_i^N x_i \times spread_i \geq return.$

The table below characterizes the input variables of the problem (without concentration limits).

Table 1: Input Parameters

	PD	spread	limit
Option A	2%	3.0%	100%
Option B	2%	7.5%	100%
Option C	7%	10.0%	100%
Option D	10%	12.0%	100%

Table 2. Results obtained (Without Concentration Limits)

	Weight A	Weight B	Weight C	Weight D	Return	VaR	CVaR	Percent
Return 6%	23.87%	26.06%	24%	26.07%	8.2%	5,0067.40	5.23	5,0067.41
Return 10%	0.01%	28.59%	35.7%	35.7%	10%	6,427.53	6.86	6,427.54

ISSN 2304-0777 www.jitbm.com

The table below shows the problem input variables (with concentration limits). Note that in this case the options B and D cannot have an investment above 10%.

Table 3. Input Parameters (With Concentration Limits)

	PD	spread	limit
Option A	2%	3.0%	100%
Option B	2%	7.5%	10%
Option C	7%	10.0%	100%
Option D	10%	12.0%	10%

Table 4. Results obtained (With Concentration Limits)

	Weight A	Weight B	Weight C	Weight D	Return	VaR	CVaR	Percent
Return 6%	40%	10%	40%	10%	7.15%	5.00	5.65	5.00
Return 10%	0%	8%	82%	10%	10%	8.2	9.08	8.2

It is worth noting in all cases the solution was found by the Excel Solver, and the time needed for resolution was always less than 1 minute. In other investment options, 10 tests were performed, the results following below.

Table 5. Input Parameters (Without Concentration Limits)

	PD	spread	limit
Option I	5%	7.5%	100%
Option II	5%	7.5%	100%
Option II	3%	4.5%	100%
Option IV	4%	6.0%	100%

ISSN 2304-0777 www.jitbm.com

Option V	5%	7.5%	100%
Option VI	6%	9.0%	100%
Option VII	7%	10.5%	100%
Option VIII	8%	12.0%	100%
Option IX	9%	13.5%	100%
Option X	10%	15.0%	100%

Table 6: Results obtained (Without Concentration Limits)

	W - I	W - II	W - III	W - IV	W - V	W - VI	W - VII	W - VIII
Return 7,5%	9,09%	9,09%	18,18%	9,09%	9,09%	9,09%	9,09%	9,09%

Table 7: Results obtained (Without Concentration Limits))

	W - IX	W - X	Return	VaR	CVaR	Percentile
Return 7,5%	9,09%	9,09%	8,86%	2,72	3,05	2,72

Where W means Weight.

Table 8. Input Parameters (With Concentration Limits)

	PD	spread	limit
Option I	5%	7.5%	100%
Option II	5%	7.5%	100%
Option II	3%	4.5%	100%

International Journal of Information Technology and Business Management
29th July 2014. Vol.27 No.1

ISSN 2304-0777 www.jitbm.com

Option IV	4%	6.0%	100%
Option V	5%	7.5%	5%
Option VI	6%	9.0%	100%
Option VII	7%	10.5%	5%
Option VIII	8%	12.0%	100%
Option IX	9%	13.5%	100%
Option X	10%	15.0%	100%

Table 9: Results obtained (Without Concentration Limits)

	W - I	W - II	W - III	W - IV	W - V	W - VI	W - VII	W - VIII
Return 7,5%	10,97%	10,90%	17,45%	11,91%	5%	10,9%	5%	10,9%

Table 10: Results obtained (Without Concentration Limits))

	W - IX	W - X	Return	VaR	CVaR	Percentile
Return 7,5%	5%	10,48%	8,73%	2,84	3,27	2,84

Where W means Weight.

6 Conclusions and Perspectives

In this paper, we propose a simple methodology for generating the distribution of credit losses through the Monte Carlo simulation. Additionally, we propose a strategy of optimizing a credit portfolio using a risk measure that considers extreme events, different from methods that use risk measures considering only normal market conditions. The formulation used has theoretical properties that facilitate the determination of optimal conditions. The addition of a simple methodology for generating the distribution of credit losses with a suitable optimization strategy in one study represents an innovation that can be easily implemented by financial institutions in Brazil.

Based on the good results obtained in this study, the authors hope to develop new techniques for the same problem addressed in a multi-objective way, seeking efficient solutions regarding two

ISSN 2304-0777 www.jitbm.com

important goals: minimizing credit risk and maximizing the return.

References

1. L.S. Abraham, Credit Scoring, Desenvolvimento, Implantação e Acompanhamento (Development, Implanting and Supervision), Editora Blucher (2010)

2. C. Acerbi, D. Tasche, Expected Shortfall. a natural coherent alternative to Value at Risk . Economic Notes, 31 2 (2002), 379-388 pp. 7-12, 1999.

3. F. Andersson, H. Mausser, D. Rosen, S.Uryasev, Credit Risk Optimization with Conditional Value-At-Risk Criterion, Mathematical Programming, Series B 89, (2001), 273-291.

4. P. Artzner, F. Delbaen, J. Eber, e D, Heath Coherent Measures of Risk. Mathematical Finance, 9 (1999), 2003-228

5. W. Bussab de O., P.A. Morettin, Estatística Básica (Basic Statistics),5 ed (5th Ed). - São Paulo. Saraiva, 2005.

6. R. T. Rockafellar and S. Uryasev, Optimization of Conditional Value-at-Risk , Journal of Risk 2 Pages. 21-41 January, 2000.

7. P.J. Schonbucher, Credit Derivatives Pricing models - Models, Pricing and Implementation, Wiley Finance Series, November, 2003.

[1] S. Uryasev, Conditional Value-at-Risk, Optimization Algorithms and Applications Financial Engineering, News January 2000 83.