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INTERACTIVE TOPSIS ALGORITHM FOR A SPECIAL TYPE OF LINEAR FRACTIONAL VECTOR OPTIMIZATION PROBLEMS

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Abstract

In this paper, TOPSIS (Technique for Order Preference by Similarity Ideal Solution) is extended for solving Linear Fractional Vector Optimization problems (LFVOP) of a special type, where such problems have block angular structure of the constraints.. In order to obtain a compromise (satisfactory) solution to the above problems using the TOPSIS approach, a modified formulas for the distance function from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS) are proposed. An interactive decision making algorithm for generating a compromise (satisfactory) solution through TOPSIS approach is provided where the decision maker (DM) is asked to specify the degree α and the relative importance of the objectives. Finally, a numerical example is given to clarify the main results developed in the paper.

Keywords: *TOPSIS, Interactive decision making, Vector optimization problems, Linear fractional programming, Bock angular structure, Angular and dual - angular structure, Fuzzy set theory, Compromise (satisfactory) solution, Poitive ideal solution, Negative ideal solution.*

1.0 INTRODUCTION

Fractional programming has attracted the attention of many researchers in the past. The main reason for interest in fractional programming stems from the fact that programming models could better fit the real problems if we consider optimization of ratio between the physical and/or economic quantities. Literature survey reveals wide applications of fractional programming in different areas ranging from engineering to economics.

There are many familiar structures for large scale optimization problems [2,3,4,5,22, 37,38, 38,47,48], such as: (i) the block angular structure, and (ii) angular and dual- angular structure to the constraints, and several kinds of decomposition methods for linear and nonlinear programming problems with those structures have been proposed in [20, 36]. An algorithm for parametric large scale integer linear multiple objective decision- making problems with block angular structure has been presented in [2].

Recently a significant number of studies have indeed been reported on single and multiple objective fractional linear and nonlinear programming problems [3, 5, 12, 14, 18, 19,20, 25,26,27,40,42,43,44,45,46,49].

Ammar [12] discussed Dinkelbach's global optimization approach for finding the global maximum of the fractional programming problem. Based on this idea, the author gave several characterizations of the solution set of a convex–concave fractional programs.

Caballero and Hernández [14] introduced a new method to estimate the weakly efficient set for the multiobjective linear fractional programming problem. A useful and new approach to solve fractional programming with absolute-value functions (FP-A) has been proposed by Chang [18]. Later on, Chang in [19] introduced an approximate approach to reaching as close as possible an optimal solution of the fractional programming problem with absolute value function (FP-A).

Dutta et al. [25] developed a method for optimizing multiobjective linear fractional programming problem which yields always an efficient solution.

Husain and Jabeen [28] derived necessary and sufficient optimality conditions for a continuous - time fractional minmax programming problem. In another work, Husain et al. [29] introduced necessary and sufficient optimality conditions for a nondifferentiable fractional minimax programming problem.

Moreover, sufficient optimality conditions for a nonlinear multiple objective fractional programming problem involving η-semidifferentiable type I-preinvex and related functions have been derived by Mishra et al. [40].

A goal programming (GP) procedure for fuzzy multiobjective linear fractional programming problems has been suggested by Pal et al. [42]. Preda [43] presented necessary and sufficient optimality conditions for a nonlinear fractional multiple objective programming problem involving η-semidifferentiable functions.

Ravi and Reddy [44] have modeled chemical process plant operations planning in an oil refinery as fuzzy linear fractional multiple goal programming problem.

Saad and Sharif [45] proposed a solution procedure to solve the chance-constrained integer linear fractional programming problem.

Stancu-Minasian and Pop [49] pointed out certain shortcomings in the work of Dutta et al. [25] and give the correct proof of theorem which validates the obtaining of the efficient solutions.

M. A. Abo–Sinna and T. H. M. Abou-El-Enien [7] introduce an algorithm for solving large scale multipe objective decision making (LSMODM) problems by use of TOPSIS.
M. A. Abo-Sinna and A. H. Amer [8] focus on multiobjective large scale nonlinear programming (MOLSNLP) problems with block angular structure. They extend TOPSIS approach to solve (MOLSNLP) problems.

G. Tzeng *et al.* [50] compare and apply TOPSIS and VIKOR to determine the best compromise alternative fuel mode. The result shows that the hybrid electric bus is the most suitable substitute bus for Taiwan urban areas in the short and median term. But, if the cruising distance of electric bus extends to an acceptable range, the pure electric bus could be the best alternative.

M. A. Abo–Sinna and T. H. M. Abou-El-Enien [9] extend TOPSIS for solving interactive large scale multiple Objective programming problems involving fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers. For such problems, the α-Pareto optimality is introduced by extending the ordinary Pareto optimality on the basis of the α-Level sets of fuzzy numbers. An interactive fuzzy decision making algorithm for generating α-Pareto optimal solution through TOPSIS approach is provided where the decision maker (DM) is asked to specify the degree α and the relative importance of objectives. Finally, a numerical example is given to clarify the main results developed in th**is** paper.

O. M. Saad and T. H. M. Abou-El-Enien [46] are concerned with the solution of integer linear fractional multiple objective programming problems (ILFMOP) of a special type, where such problems have block angular structure of the constraints. The presented solution algorithm is based mainly upon a combination of Charens - Cooper transformation coupled with Branch and Bound method together with Rosen's Partitioning procedure. To demonstrate the proposed algorithm, a numerical example to clarify the main results and the developed theory in the paper is provided

T. H. M. Abou-El-Enien and O. M. Saad [3] are concerned with solving large scale linear fractional multiple objective programming (LSLFMOP) problems with chance constraints. Channce constraints involve random parameters in the right-hand sides. These random right-hand sides are considered to be statistically independent random variable. The main features of the proposed solution procedure are based on chance-constrained technique, Charens and Cooper transformation and the Rosen's partitioning procedure. An illustrative

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numerical example is given to clarify the main results developed in the paper.

T. H. M. Abou-El-Enien [4] focuses on the solution of a Large Scale Integer Linear Vector Optimization Problems with chance constraints (CHLSILVOP) of a special type through the Technique for Order Preference by Similarity Ideal Solution (TOPSIS) approach, where such problems has block angular structure of the constraints. Chance constraints involve random parameters in the right hand sides. TOPSIS is extended to solve CHLSILVOP with constraints of block angular structure. Compromise (TOPSIS) control minimizes the measure of distance, provided that the closest solution should have the shortest distance from the positive ideal solution (PIS) as well as the longest distance from the negative ideal solution (NIS). As the measure of "closeness" d_P-metric is used. Thus, He reduces a k-dimensional objective space to a two-dimensional space by a first-order compromise procedure. The concept of a membership function of fuzzy set theory is used to represent the satisfaction level for both criteria. Moreover, He derives a single objective large scale Integer programming problem using the max–min operator for the second-order compromise operation. Also, An interactive decision making algorithm for generating integer Pareto optimal (compromise) solution for CHLSILVOP through TOPSIS approach is provided where the decision maker (DM) is asked to specify the relative importance of objectives. Finally, a numerical example is given to clarify the main results developed in this paper.

After the publication of TOPSIS approach [30,34], the subsequent works in this area of optimization have been numerous (see f. i. [1,4,5,6,7,8,9,10,11,15,16,17,24,31,32,33,34,35, 50]. In the present paper, a solution algorithm is suggested for the solution of linear fractional vector optimization problems (LFVOP) with block angular structure of the constraints through TOPSIS approach.

The paper is organized as follows: In the following section, the problem formulation of LFVOP which has block angular structure is formulated. An algorithm is described in finite steps for solving the problem of concern is proposed in Section 3. For the sake of illustration, a numerical example is provided in Section 4. Finally, the paper is concluded in Section 5.

2.0 PROBLEM FORMULATION

Consider the following linear fractional vector optimization problem with a block angular structure of the constraints as [3,46]**:**

$$\textit{Minimize} \quad (F_1(X), F_2(X), \ldots\ldots, F_k(X)) \qquad (1\text{-a})$$

subject to

$$X \in M = \left\{ \begin{array}{c} X \in R^n \,|\, \sum_{j=1}^{q} A_j X_j \le v_\circ, \\ B_j X_j \le v_j, \\ X_j \ge 0\,, j = 1,2,\ldots, q, q > 1 \end{array} \right\} \qquad (1\text{-b})$$

where the i^{th} objective function can be written as follows:

$$F_i(X) = \frac{\sum_{j=1}^{q} f_{ij}(X)}{\sum_{j=1}^{q} f_j(X)} = \frac{\sum_{j=1}^{q} C_{ij} X_j + \gamma_i}{\sum_{j=1}^{q} D_j X_j + \beta}\ , \ i=1,\ 2, \ldots\ldots, k \quad (2)$$

and

γ_i and β are constants, $i = 1,2,\ldots,k;$

k : the number of objective functions,

q : the number of subproblems,

m : the number of constraints,

n : the number of variables,

n_j : the number of variables of the j^{th} subproblem,

$j=1,2,\ldots,q,\ q>1$

m_o : the number of the common constraints

represented by: $\sum_{j=}^{q} A_j X_j \le v_0,$

m_j : the number of independent constraints of the j^{th}

subproblem represented by:

$B_j X_j \le v_j,\quad j=1,2,\ldots,q,\ q>1$

R : the set of all real numbers,

X : an n-dimensional column vector of variables,

X_j : an n_j-dimensional column vector of variables for the j^{th} subproblem, $j=1,2,....,q,\ q>1$,

A_j : an $(m_o \times n_j)$ coefficient matrix,

B_j : an $(m_j \times n_j)$ coefficient matrix,

v_o : an m_o-dimensional column vector of right-hand sides of the common constraints whose elements are constants,

v_j : an m_j-dimensional column vector of independent constraints right-hand sides whose elements are the constants of the constraints for the j^{th} subproblem, $j=1,2,...,q,\ q>1$.

C_{ij} : an n_j-dimensional row vector for the j^{th} subproblem in the i^{th} objective function, D_{ij} : an n_j-dimensional row vector for the j^{th} subproblem in the i^{th} objective function,

$K = \{1,2,....,k\}$, $N = \{1,2,...,n\}$ and $R^n = \{X=(x_1, x_2,...,x_n\}^T : x_i \in R,\ i \in N\}$.

Furthermore, we assume that $\sum_{j=1}^{q} D_j X_j + \beta$ is everywhere positive on the set of non-negative numbers.

Consequently, using Charnes-Cooper transformation method [20] by making the variable change:

$$\mu = \frac{1}{\sum_{j=1}^{q} D_j X_j + \beta_j} \tag{3-a}$$

with the additional variable changes

$$Y_j = X_j \mu,\ j = 1,2,\dots,q, q > 1 \tag{3-b}$$

then under these changes, problem (1) is equivalent to the following problem:

$$Minimize\ \ (F_1(Y,\mu), F_2(Y,\mu), \dots\dots, F_k(Y,\mu)) \tag{4-a}$$

subject to

$$X \in M' = \left\{ \begin{array}{r} X \in R^n \,|\, \sum_{j=1}^{q} A_j Y_j - v_\circ \mu \le 0, \\ \sum_{j=1}^{q} D_j Y_j + \beta\mu = 1 \\ B_j Y_j - v_j \mu \le 0, \\ \mu > 0, \\ Y_j \qquad \ge 0, \\ j = 1,2,\dots,q, q > 1 \end{array} \right\} \tag{4-b}$$

where equation (2) can be rewritten as follows :

$$F_i(X) = F_i(Y,\mu) = \sum_{j=1}^{q} F_{ij}(Y,\mu)$$

$$= \sum_{j=1}^{q} C_{ij} y_j + Y_j \mu, \quad i = 1,2,\dots,k,$$

Now, problem (4-a)-(4-b) has angular and dual-angular structure [21,37] and can be solved using the Rosen's partitioning procedure [21,37] to find its optimal solution Y_j^*, which will give the optimal solution $X_j^* = Y_j^* / \mu^*$ to problem (1).

3.0 SOME BASIC CONCEPTS OF DISTANCE MEASURES

The compromise programming approach [5,26,30,31,34,35, 52] has been developed to perform multiple objective decision making problems, reducing the set of nondominated solutions [50]. The compromise solutions are those which are the closest by some distance measure to the ideal one.

The point $F_i(X^*) = \sum_{j=1}^{q} F_{ij}(X^*)$ in the criteria space is called the ideal point (reference point). As the measure of "closeness", d_p-metric is used. The d_p-metric defines the distance between two points, $F_i(X) = \sum_{j=1}^{q} F_{ij}(X)$ and $F_i(X^*) = \sum_{j=1}^{q} F_{ij}(X^*)$ (the reference point) in k-dimensional space [40] as:

$$d_p = \left(\sum_{i=1}^{k} (F_i^* - F_i)^p \right)^{\frac{1}{p}}$$

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$$= \left(\sum_{i=1}^{k} \left(\sum_{j=1}^{q} F_{ij}^{*} - \sum_{j=1}^{q} F_{ij} \right)^{p} \right)^{\frac{1}{p}} \qquad (5)$$

where p≥1.

Unfortunately, because of the incommensurability among objectives, it is impossible to directly use the above distance family. To remove the effects of the incommensurability, we need to normalize the distance family of equation (5) by using the reference point [5,26,30,31,34,35,52]as :

$$d_p = \left(\sum_{i=1}^{k} \left(\frac{\sum_{j=1}^{q} F_{ij}^{*} - \sum_{j=1}^{q} F_{ij}}{\sum_{j=1}^{q} F_{ij}^{*}} \right)^{p} \right)^{1/p} \qquad (6)$$

where $p \geq 1$.

To obtain a compromise solution for problem (1), the global criteria method [30, 53] for large scale problems uses the distance family of equation (5) by the ideal solution being the reference point. The problem becomes how to solve the following auxiliary problem :

$$\underset{x \in M}{Minimize}\ d_p = \left(\sum_{i=1}^{k} \left(\frac{\sum_{j=1}^{q} F_{ij}(X^{*}) - \sum_{j=1}^{q} F_{ij}(X)}{\sum_{j=1}^{q} F_{ij}(X^{*})} \right)^{p} \right)^{1/p} \qquad (7)$$

where X^{*} is the PIS and $p = 1,2, \ldots\ldots, \infty$.

Usually, the solutions based on PIS are different from the solutions based on NIS. Thus, both $PIS(F^{*})$ and $NIS(F^{-})$ can be used to normalize the distance family and obtain [5] :

$$d_p = \left(\sum_{i=1}^{k} \left(\frac{\sum_{j=1}^{q} F_{ij}^{*} - \sum_{j=1}^{q} F_{ij}}{\sum_{j=1}^{q} F_{ij}^{*} - \sum_{j=1}^{q} F_{ij}^{-}} \right)^{p} \right)^{1/p} \qquad (8)$$

where $p \geq 1$.

In this study, the concept of TOPSIS is extended to obtain a Pareto Optimal (compromise) solution for LFVOP problms.

4.0 TOPSIS FOR LFVOP:

Consider the following LFVOP [5]:

$$Maximize/_{Minimize}\ (F_1(X), F_2(X), \ldots\ldots, F_k(X)) \qquad (9-a)$$

subject to

$$X \in M^{/} \qquad (9-b)$$

where

$\sum_{j=1}^{q} F_{tj}(X)$: Objective Function for Maximization, $t \in K_1 \sqsubset K$,

$\sum_{j=1}^{q} F_{vj}(X)$: Objective Function for Minimization, $v \in K_2 \sqsubset K$.

In order to use the distance family of equation (8) to resolve problem (9), we must first find $PIS(F^{*})$ and $NIS(F^{-})$ which are [5]:

$$F^{*} = \underset{x \in M^{/}}{Maximize(or\ Minimize)} \sum_{j=1}^{q} F_{tj}(X) \left(or \sum_{j=1}^{q} F_{vj}(X) \right),$$

$$\forall t (and\ v) \qquad (10-a)$$

$$F^{-} = \underset{x \in M^{/}}{Minimize(or\ Maximize)} \sum_{j=1}^{q} F_{tj}(X) \left(or \sum_{j=1}^{q} F_{vj}(X) \right),$$

$$\forall t (and\ v) \qquad (10-b)$$

where $K = K_1 \cup K_2$.

$F^{*} = (F_1^{*}, F_2^{*}, \ldots., F_k^{*})$ and $F^{-} = (F_1^{-}, F_2^{-}, \ldots\ldots, F_k^{-})$ are the individual positive(negative) ideal solutions.

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Using the PIS and the NIS, we obtain the following distance functions from them, respectively:

$$d_p^{PIS} = \left(\sum_{t \in k_1} w_t^p \left(\frac{\sum_{j=1}^q F_{tj}^* - \sum_{j=1}^q F_{tj}(X)}{\sum_{j=1}^q F_{tj}^* - \sum_{j=1}^q F_{tj}^-} \right)^p + \sum_{v \in k_2} w_v^p \left(\frac{\sum_{j=1}^q F_{vj}(X) - \sum_{j=1}^q F_{vj}^*}{\sum_{j=1}^q F_{vj}^- - \sum_{j=1}^q F_{vj}^*} \right)^p \right)^{1/p} \quad (11\text{-a})$$

and

$$d_p^{NIS} = \left(\sum_{t \in k_1} w_t^p \left(\frac{\sum_{j=1}^q F_{tj}(X) - \sum_{j=1}^q F_{tj}^-}{\sum_{j=1}^q F_{tj}^* - \sum_{j=1}^q F_{tj}^-} \right)^p + \sum_{v \in k_2} w_v^p \left(\frac{\sum_{j=1}^q F_{vj}^- - \sum_{j=1}^q F_{vj}(X)}{\sum_{j=1}^q F_{vj}^- - \sum_{j=1}^q F_{vj}^*} \right)^p \right)^{1/p} \quad (11\text{-b})$$

where $w_i = 1,2, \ldots, k$, are the relative importance (weights) of objectives, and $p = 1,2, \ldots, \infty$.

In order to obtain a compromise solution, transfer problem (9) into the following bi-objective problem with two commensurable (but often conflicting) objectives [5]:

$Minimize\ d_p^{PIS}(X)$

$Maximize\ d_p^{NIS}(X)$

subject to (12)

$X \in M^/$

where $p = 1,2, \ldots, \infty$.

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions ($\mu_1(X)$ and $\mu_2(X)$) of two objective functions are linear. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the PIS for $\mu_1(X)$ and assign a larger degree to the one with farther distance from NIS for $\mu_2(X)$. Therefore, as shown in figure (1), $\mu_1(X)$ and $\mu_2(X)$ can be obtained as the following [5,54]:

$$\mu_1(X) = \begin{cases} 1, & if\ d_p^{PIS}(X) < \left(d_p^{PIS}\right)^*, \\ 1 - \dfrac{d_p^{PIS}(X) - \left(d_p^{PIS}\right)^*}{\left(d_p^{PIS}\right)^- - \left(d_p^{PIS}\right)^*}, & if\ \left(d_p^{PIS}\right)^- \ge d_p^{PIS} \ge \left(d_p^{PIS}\right)^*, \\ 0, & if\ d_p^{PIS}(X) > \left(d_p^{PIS}\right)^-, \end{cases} \quad (13\text{-a})$$

$$\mu_2(X) = \begin{cases} 1, & if\ d_p^{NIS}(X) > \left(d_p^{NIS}\right)^*, \\ 1 - \dfrac{d_p^{NIS}(X) - \left(d_p^{NIS}\right)^-}{\left(d_p^{NIS}\right)^* - \left(d_p^{NIS}\right)^-}, & if\ \left(d_p^{NIS}\right)^- \le d_p^{NIS} \le \left(d_p^{NIS}\right)^*, \\ 0, & if\ d_p^{NIS}(X) < \left(d_p^{NIS}\right)^-, \end{cases} \quad (13\text{-b})$$

where

$\left(d_p^{PIS}\right)^* = \underset{x \in M^/}{Minimize}\, d_p^{PIS}(X)$ and the solution is X^{PIS},

$\left(d_p^{NIS}\right)^* = \underset{x \in M^/}{Maximize}\, d_p^{NIS}(X)$ and the solution is X^{NIS},

$\left(d_p^{PIS}\right)^- = d_p^{PIS}(X^{NIS})$ and $\left(d_p^{NIS}\right)^- = d_p^{NIS}(X^{PIS})$.

Now, by applying the max-min decision model which is proposed by Bellman and Zadeh [13] and extended by Zimmermann [54], we can resolve problem (12). The satisfying decision, X^*, may be obtained by solving the following model :

$$\mu_D(X^*) = \underset{X \in M^/}{Maximize} \left\{ Min.\left(\mu_1(X),\ \mu_2(X)\right) \right\} \quad (14)$$

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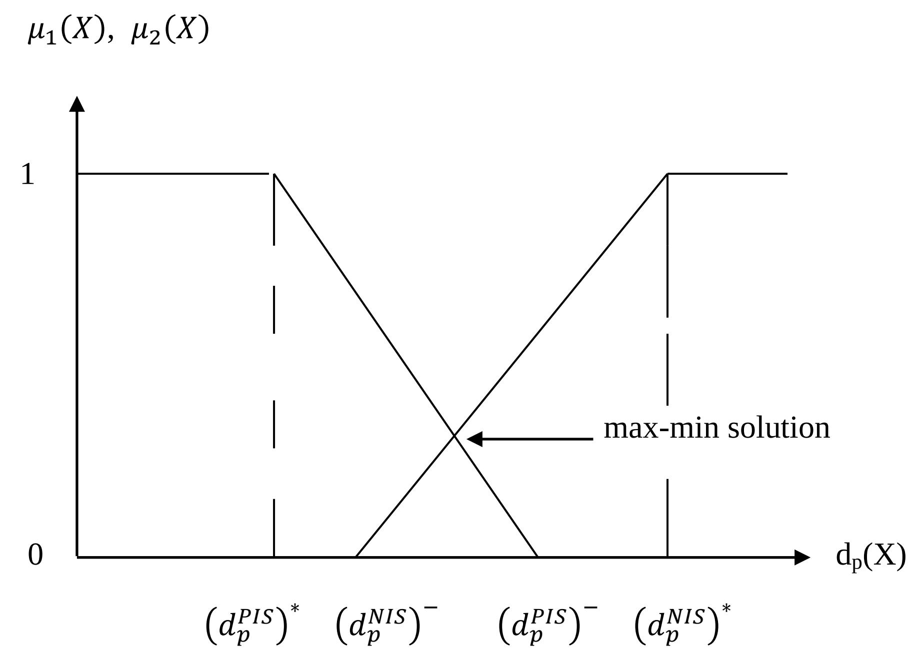

Figure 1. The membership functions of $\mu_1(X)$ and $\mu_2(X)$

Finally, if $\delta = Min.(\mu_1(X),\ \mu_2(X))$, the model (14) is equivalent to the form of Tchebycheff model [23], which is equivalent to the following model:

$$Maximize\ \delta, \qquad (15-a)$$

subject to

$$\mu_1(X) \geq \delta, \qquad (15-b)$$

$$\mu_2(X) \geq \delta, \qquad (15-c)$$

$$X \in M^{/}, \quad \delta \in [0,1] \qquad (15-d)$$

where δ is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

5.0 THE ALGORITHM OF TOPSIS FOR SOLVING LFVOP:

Thus, we can introduce the following algorithm to gernerate a set of Pareto Optimal (compromise) solutions for the LFVOP:

The algorithm (Alg-I):

Step 1. Formulate LFVOP (1) which has linear fractional objective functions as in Eq. (2) and block angular structure.

Step 2. Use the Charnes-Cooper transformation [20] by making the variable change $\mu = \frac{1}{\sum_{j=1}^{q} D_j X_j + \beta_j}$ with the additional variable change $y_j = X_j\, \mu$ j=1, 2, ….., q, q>1 to rewrite problem (1) in the form of problem (4)

Step 3. Transform problem (4) to the form of problem (9)

Step 4. Construct the PIS payoff table of problem (9) by using Rosen's partitioning procedure [21, 37]. Thus, $F^* = (F_1^*, F_2^*, \ldots, F_k^*)$, the individual positive ideal solutions are obtained.

Step 5. Construct the NIS payoff table of problem (9) by using Rosen's partitioning procedure [21, 37]. Thus, $F^- =$

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$(F_1^-, F_2^-, \ldots\ldots, F_k^-)$, the individual negative ideal solutions are obtained.

Step 6. Use equations (10 & 11) and steps (4 & 5) to construct d_p^{PIS} and d_p^{NIS}.

Step 7. Transform problem (9) to problem (12).

Step 8. Ask the DM to select $p = p^* \in \{1,2,\ldots\ldots,\infty\}$,

Step 9. Ask the DM to select $w_i = w_i^*, i = 1,2,\ldots,k$, where $\sum_{i=1}^{k} w_i = 1$

Step 10. Construct the payoff table of problem (12):

At p=1, use Rosen's partitioning procedure [21, 37].

At p≥2, use the generalized reduced gradient method [37, 38] and obtain:

$$d_p^- = \left(\left(d_p^{PIS}\right)^-, (d_P^{NIS})^-\right),$$

$$d_p^* = \left(\left(d_p^{PIS}\right)^*, (d_P^{NIS})^*\right),$$

Step 11. Construct problem (15) by the use of the membership functions (13).

Step 12. Solve problem (15).

Step 13. If the solution of problem (15) yields optimal solution, then go to step 14. Otherwise go to step 15.

Step 14. If the DM is satisfied with the current solution, go to step 15. Otherwise, go to step 8.

Step 15. Stop.

6. 0 AN ILLUSTRATIVE NUMERICAL EXAMPLE

In what follows, we provide a numerical example to illustrate the solution algorithm described in the previous section. For this purpose, let us consider the following linear fractional vector optimization problem which has a block angular structure of the constraints as:

$$\begin{aligned} &\textit{Maximize } F_1(x)\\ &\textit{Minimize } F_2(x),\\ &\text{subject to}\\ &\quad x_1 + x_2 \le 4,\\ &\quad x_1 \le 3,\\ &\quad x_1 \ge 1,\\ &\quad x_2 \le 1.5,\\ &\quad x_2 \ge .0.5,\\ &\quad x_1, x_2 \ge 0, \end{aligned} \tag{16}$$

where

$$F_1(x) = \frac{3x_1 + 5x_2}{4x_1 + 3x_2 + 3}, \quad \text{and}$$

$$F_2(x) = \frac{7x_1 + 2x_2}{4x_1 + 3x_2 + 3}$$

Assume that $\mu = \dfrac{1}{4x_1 + 3x_2 + 3}$,

therefore the above problem will take the following form:

$$\begin{aligned} &\textit{Maximize } F_1(x,\mu)\\ &\textit{Minimize } F_2(x,\mu)\\ &\text{subject to}\\ &\quad x_1\mu + x_2\mu \le 4\mu,\\ &\quad 4x_1\mu + 3x_2\mu + 3\mu = 1,\\ &\quad x_1\mu \le 3\mu, \quad x_1\mu \ge \mu,\\ &\quad x_2\mu \le 1.5\mu, \; x_2\mu \ge 0.5\mu,\\ &\quad x_1, x_2 \ge 0, \; \mu > 0. \end{aligned} \tag{17}$$

Now, making the variable changes:

$$y_1 = x_1\mu \text{ and } y_2 = x_2\mu$$

Thus, the above problem can be rewritten as follows:

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$$\begin{aligned}&Maximize\ \ F_1(y,\mu)\\&Minimize\ \ F_2(y,\mu)),\\&\text{subject to} \qquad\qquad\qquad (18)\\&y_1+y_2-4\mu\le 0,\\&4y_1+3y_2+3\mu=1,\\&y_1-3\mu\le 0,\quad y_1-\mu\ge 0,\\&y_2-1.5\mu\le 0,\quad y_2-0.5\mu\ge 0,\\&y_1,y_2\ge 0\ \ and\ \ \mu>0,\end{aligned}$$

This problem has angular and dual-angular structure.

Table (1): PIS payoff table of problem (18)

	f_1	f_2	y_1	y_2	μ
Maximize $f_1(X)$	0.91304*	0.86957	0.08696	0.13043	0.08696
Minimize $f_2(X)$	0.91304	0.86957*	0.08696	0.13043	0.08696

PIS: f^*=(0.91304, 0.86957)

Table (2): NIS payoff table of problem (16)

	f_1	f_2	y_1	y_2	μ
Minimize $f_1(X)$	0.64706-	094119	0.11765	0.05882	0.11765
Maximize $f_2(X)$	0.6971	1.33333-	0.18182	0.0303	0.06061

NIS: f^-=(0.64706, 1.33333)

Next, we compute equation (11) and obtain the following equations:

$$d_p^{PIS} = \left[w_1^p\left(\frac{0.91304-f_1(X)}{0.91304-0.64706}\right)^p + w_2^p\left(\frac{f_2-0.86957}{1.33333-0.86957}\right)^p\right]^{1/p}$$

$$d_p^{NIS} = \left[w_1^p\left(\frac{f_1-0.64706}{0.91304-0.64706}\right)^p + w_2^p\left(\frac{1.33333-f_2(X)}{1.33333-0.86957}\right)^p\right]^{1/p}$$

Thus, problem (12) is obtained.

In order to get numerical solutions, let us assume that $w_1^p = w_2^p = 0.5$ and (1) at p=1,

Table (3): PIS payoff table of problem (12), when p=2,

	d_1^{PIS}	d_1^{NIS}	y_1	y_2	μ
Minimize d_2^{PIS}	0.000004*	0.7071-	0.087	0.1304	0.087
Maximize d_2^{NIS}	0.000004-	0.7071*	0.087	0.1304	0.087

d_1^*= (0.000004, 0.7071) , d_1^-= (0.000004, 0.7071)

Now, it is easy to use problem (15) to formulate the following problem:

$$Maximize\ \ \delta$$

Subject to:

$X \in M', \delta \in [0,1]$, $d_2^{PIS} - 0.000004 \ge 0$,

$0.7071 - d_2^{NIS} \ge 0$

The maximum "satisfactory level" ($\delta = 1$) is achieved for the solution when

y_1=0.087, y_2=0.1304, μ=0. 087, x_1= 1 and x_2=1.49885 .

7. 0 CONCLUSIONS

In this paper, a TOPSIS approach has been extended to solve LFVOP. The LFVOP using TOPSIS approach provides an effective way to find the compromise (satisfactory) solution of such problems. Generally TOPSIS, provides a broader principle of compromise for solving Vector Optimization Problems. It transfers k-objectives (criteria), which are conflicting and non-commensurable, into two objectives (the shortest distance from the PIS and the longest distance from the NIS), which are commensurable and most of time conflicting . Then, the bi-objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS, compromise solution by a second–order compromise. The max-min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS).

Also, in this paper, an algorithm of generating Pareto Optimal (compromise) solutions of LFVOP problem has been presented. It is based on the decomposition algorithm of LFVOP with block angular structure via TOPSIS approach for p=1 and Generalized reduced gradient method for p≥2 . This algorithm combines LFVOP and TOPSIS approach to obtain TOPSIS's compromise solution of the problem, Finally, a numerical illustrative example clarified the various aspects of both the solution concept and the proposed algorithm.

REFERENCES

1. Abou -El-Enien T. H. M. and Abo-Sinna M. A., "Extension of TOPSIS for Large Scale Multiple Objective Decision Making Problems under Fuzzy environment", in *the proceeding of the Second International Conference on Informatics and Systems, "INFOS 2004"*, Faculty of Computers & Information, Cairo University, March 6-8, pp. 153-170, *Egypt*, (2004).
2. Abou-El-Enien T. H. M. "An Algorithm for Parametric Large Scale Integer Linear Multiple Objective Decision Making Problems", *Advances in Modelling and Analysis*, (Series A), Vol. 42, No.4, pp.51-70, (*AMSE periodicals, France*), (2005).
3. Abou-El-Enien T. H. M. and Saad O. M. On the Solution of a Special Type of Large Scale Linear Fractional Multiple Objective Programming Problems with Uncertain Data, *Applied Mathematical Sciences*, Vol. 4, No. 62, pp. 3095–3105, (*Hikari, Bulgaria*), (2010).
4. Abou-El-Enien T. H. M." On the solution of a Special Type of Large Scale Integer Linear Vector Optimization Problems with Uncertain Data through TOPSIS approach, *International Journal of Contemporary Mathematical Sciences*, Vol. 6, No. 14, pp. 657 – 669, (*Hikari, Bulgaria*), (2011).
5. Abou-El-Enien, T. H. M. TOPSIS Algorithms for Multiple Objectives Decision Making : Large Scale Programming Approach, *LAP LAMBERT Academic Publishing*, Germany, (2013).
6. Abo-Sinna M. A., Extensions of the TOPSIS for multiobjective Dynamic Programming Problems under Fuzziness, *Advances in Modelling & Analysis*, (Series B), Vol. 43, No. 4, pp. 1-24, (*AMSE periodicals, France*), (2000).
7. Abo–Sinna M.A., and Abou-El-Enien T. H. M., An Algorithm for Solving Large Scale Multiple Objective Decision Making Problems Using TOPSIS Approach, *Advances in Modelling and Analysis*, (Series A), Vol. 42, No.6, pp. 31-48, *(AMSE periodicals, France*), (2005).
8. Abo-Sinna M. A.and Amer A. H., Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems, *Applied Mathematics and Computations*, Vol.162, pp. 243-256, (2005).
9. Abo–Sinna M.A.and Abou-El-Enien T. H. M. An Interactive Algorithm for Large Scale Multiple objective Programming Problems with Fuzzy Parameters through TOPSIS approach, *Applied Mathematics and Computation*, Vol. 177, No. 2, pp. 515-527, (*Elsevier, Holland*), (2006).
10. Abo-Sinna M. A., Amer A. H. and Ibrahim A. S., Extensions of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure, *Applied Mathematical Modelling*, 32(3) (2008), 292-302.
11. Afshar A., Mariño M. A., Saadatpour M.and Afshar A., Fuzzy TOPSIS Multi-Criteria Decision Analysis Applied to Karun Reservoirs System, *Water Resour Manage* , 25 (2011) 545–563.
12. Ammar E.E., On solutions and duality of nonlinear nonsmooth fractional, programs, *Applied Mathematics and Computation*, Vol. 172, No. 2, pp. 865-875, 2006.
13. Bellman R.E. & Zadeh L.A., Decision-making in fuzzy environment, *Management Science*, B 17(1970) 141-164,

ISSN 2304-0777 www.jitbm.com

14. Caballero R.and Hernández M., The controlled estimation method in the multiobjective linear fractional problem, *Computers & Operations Research* Vol. 31, No. 11, pp. 1821-1832, 2004.
15. Chen C.T., Extensions of the TOPSIS for group decision-making under environment, *Fuzzy Sets and Systems,* 114 (2000) 1-9.
16. Chen M. and Tzeng G., Combining grey relation and TOPSIS concepts for selecting an expatriate host country, *Mathematical and Computer Modelling,* 40(2004) 1473-1490.
17. Cheng C., Zhao M., Chau K.W. and Wu X., Using genetic algorithm and TOPSIS for Xinanjiang model calibration with a single procedure, *Journal of Hydrology,* 316(2006), 129-140.
18. Chang C., An approximate approach for fractional programming with absolute-value functions, *Applied Mathematics and Computation*, Vol. 161, No.1, pp. 171-179, 2005.
19. Chang C., Fractional programming with absolute – value functions: a fuzzy goal programming approach, *Applied Mathematics and Computation*, Vol. 167, No. 1, pp. 508-515, 2005.
20. Charnes A.and Cooper W. W., Programming with linear fractional criteria, *Naval Res. Logist Quart*., Vol. 9, pp. 181 – 186, 1962.
21. Conejo A. J., Castillo E., Minguez R.and Garcia-Bertrand R., Decomposition Techniques in Mathematical Programming, *Springer-Verlag*, *Germany*, (2006).
22. Dantzig G., & Wolfe P., The Decoposition Algorithm for Linear Programming, *Econometric*, 9(4)(1961), 1-9.
23. Dauer J.P., and.Osman M.S.A, Decomposition of the Parametric Space in Multiobjective Convex Programs using the Generalized Tchebycheff Norm, *Journal of Mathematical Analysis and Applications*, 107(1) (1985), 156-166.
24. Deng H., Yeh C.H.& Willis R. J., Inter-company comparison using modified TOPSIS with objective weights, *Computers & Operations Research,* 17(2000), 963-973.
25. Dutta D., Tiwari R. N. and Rao J. R., Multiple objective linear fractional programming - A fuzzy set theoretic approach, *Fuzzy Sets and Systems* Vol. 52, No. 1, pp. 39-45, 1992.
26. Freimer M., and Yu P. L., Some New Results on Compromise Solutions for Group Decision Problems, *Management Science*, Vol. 22, No. 6, pp. 688-693, (1976).
27. Ho J. K., and Sundarraj R. P., Computational Experience with Advanced Implementation of Decomposition Algorithm for Linear Programming , *Mathematical Programming*, 27 (1983), 283-290.
28. Husain I. and Jabeen Z., Continuous-time fractional minmax programming, *Mathematical and Computer Modelling*, Vol. 42, No. (5-6), pp. 701-710, 2005.
29. Husain I., Hanson M. A. and Jabeen Z., On nondifferentiable fractional minimax programming, *European Journal of Operational Research*, Vol. 160, No.1, pp. 202-217, 2005.
30. Hwang C.L., & Masud A.S. M., Multiple Objective Decision Making Methods and Applications, Springer-Verlag, New York, USA, (1979).
31. Hwang C. L. and Yoon K., Multiple Attribute Decision Making Methods and Applications: A State-of-the-Art-Survey, Lecture Notes in Economics and Mathematical Systems 186, Springer-Verlag, Berlin, Heidelberg, (1981).
32. Joshi D. & Kumar S., Intuitionistic fuzzy entropy and distance measure based TOPSIS method for multi-criteria decision making, *Egyptian Informatics Journal*, Vol. 15, pp. 97-104, 2014.
33. Kahraman C. (ed), Fuzzy Multi-Criteria Decision Making: Theory and Applications with Recent Developments, Springer, New York, (2008).
34. Lai Y. J. and Hwang C.L., Fuzzy Multiple Objective Decision Making: Methods and Applications, Springer-Verlag,Heidelbeg, (1994).
35. Lai Y. J., Liu T.Y. & Hwang C.L., TOPSIS for MODM, *European Journal of Operational Research*, 76(1994), 486-500.
36. Lai Y. J., Hierarchical optimization: A satisfactory solution, *Fuzzy Sets and Systems*, 77 (1996), 321–335.
37. Lasdon L. S., Optimization theory for large systems, Macmillan, New York, 1970.
38. Lasdon L. S., Waren A. D., &.Ratner M.W, GRG2 User's Guide Technical Memorandum, University of Texas, (1980).
39. Martin R. K., Large scale linear and integer optimization, Springer, U.S.A., (1999).
40. S.K. Mishra, Wang S.Y. and Lai K.K., Multiple objective fractional programming involving semilocally type I-preinvex and related functions, *Journal of Mathematical Analysis and Applications*, Vol. 310, No. 2, pp. 626-640, 2005.
41. Nikolsky S. M., " A Course of Mathematical Analysis", Mir Publishers, Moscow, USSR, (1987).
42. Pal B. B., Moitra B. N. and Maulik U., A goal programming procedure for fuzzy multiobjective linear fractional programming problem, *Fuzzy*

ISSN 2304-0777 www.jitbm.com

Sets and Systems Vol. 139, No. 2, pp. 395-405, 2003.

43. Preda V., Optimality and duality in fractional multiple objective programming involving semilocally preinvex and related functions, *Journal of Mathematical Analysis and Applications*, vol. 288, No. 2, pp. 365-382, 2003.

44. Ravi V. and Reddy P. J., Fuzzy linear fractional goal programming applied to refinery operations planning, *Fuzzy Sets and Systems*, Vol. 96, No.2, pp. 173-182, 1998.
45. Saad O. M. and Sharif W.H., On the solution of integer linear fractional programs with uncertain data, *Journal of the Mathematics & Computer Sciences*, Vol. 12, No.2, pp. 169-173, 2001,
46. Saad O. M. and Abou-El-Enien T. H. M. , On the solution of a special type of integer linear fractional multiple objective programming problems, Advances in Modelling and Analysis, (Series D), Vol. 13, No.1, (AMSE periodicals, France), 2008.
47. Sakawa M., Large Scale Interactive Fuzzy Multiobjective Programming, Physica-Verlag, A Springer-Verlag, New York, (2000)
48. Smelyanskiy M. and Skedzielewski S., Parallel computing for large-scale optimization problems: Challenges and Solutions, *Intel Technology Journal*, Vol. 9, No. 2, 151-164, 2005.

49 Stancu-Minasian. I. M. and Pop B., On a fuzzy set approach to solving multiple objective linear fractional programming problem, *Fuzzy Sets and Systems*, Vol. 134, No. 3, pp. 397-405, 2003.

50. Tzeng G., Lin C. and Opricovic S., Multi-criteria analysis of alternative-fuel buses for public transportation, *Energy Policy*, 33(2005), 1373-1383.

51. Yu P. L. and Zeleny M., The set of all non-dominated solutions in linear cases and a multicriteria decision making, *Journal of Mathematical Analysis and Applications*, 49(1975),430-448.
52. Zeleny M., Compromise Programming, in:J.L. Cochrane and M. Zeleny (eds.), Multiple Criteria Decision Making, University of South Carolina, Columbia, SC, (1973), 262-300.

53. Zeleny M., Multiple Criteria Decision Making, McGraw-Hill, New York, 1982.

54. Zimmermann H.-J., Fuzzy Sets, Decision Making, and Expert Systems, Kluwer Academic Publishers, Boston, 1987.